ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

Supply Quota Allocation and its optimization in supply chain by Fuzzy programming Model

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Abstracts

Appropriate vendor selection and their optimum quota allocation has been an area of high importance in the effective management of a supply chain. The optimization of vendor-base is needed to identify better performing vendors in a supply chain. Secondly their quota allocations need to be optimized for the organization to remain competitive in the global scenario.

The Supplier Quota Allocation (SQA) concept 'Multi-Objective Linear Programming Vendor Selection Problem the mathematical formulation of which incorporates three important goals- cost- minimization, rejection-minimization (or quality-maximization) and minimization of late-delivery- with practical constraints imposed on: meeting the manufacturing organization's annual aggregate demand, vendors' capacity, vendors' quota flexibility, vendors' rating etc. In FP formulation of a supply chain modeling, various input parameters are treated as vague with a linear membership function of fuzzy type.

Keywords: supply chain management, fuzzy programming, LINDO, MATLAB.

Nomenclature
AHP : Analytical Hierarchy Process
DSS : Decision Support System
JIT : Just In Time.
MCD : Multiple Criteria Decision
QC : Quality Control
SC : Supply Chain
SCM : Supply Chain Management
VSP : Vendor Selection Problem
VDP : Vendor Development Program

Model Specific
FGP : Fuzzy Goal Programming
SOLP : Single Objective Linear Programming
SQA : Supplier Quota Allocation

Introduction

The supply chain of a manufacturing organization consists of all the activities, which are required to deliver the products and services to the end customers. It includes receiving orders from customers through marketing division, procuring raw materials from vendors, manufacturing, and logistics in man-machine-material management, marketing, customer relations and so on. The effective integration of information and material flow within the demand and supply process is what supply chain management is all about. 'The objective of managing the supply chain is to synchronize the requirements of the customers with the flow of materials from suppliers in order to strike a balance between what are often seen as conflicting goals of high customer service, low inventory, and low unit cost'. Each node in a supply chain is a strategic link. The strong links make strong supply chains while the weak links hurt every member of the chain.

Appropriate vendor selection and their optimum quota allocation has been an area of high importance in the effective management of a supply chain. This is due to

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
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the compelling need to develop long-term strategic alliances with the vendors, because material and equipment supplied by the vendors play an important role in the management of a supply chain of any organization. Many other issues in the supply chain are directly or indirectly influenced by the proper selection of vendors. Having a policy of approved reliable vendors may lead to less number of vendors in a supply chain, whereas the selection of a large number of vendors may be done to minimize the risk associated with the purchase. Hence, the optimization of vendor-base is needed to identify better performing vendors in a supply chain. Secondly their quota allocations need to be optimized for the organization to remain competitive in the global scenario.

Literature review

Muralidharan, Anantharaman, & Deshmukh, (2002) - The operations of all the organizations within the supply chain depend on the selection of suppliers that align with the goals and mission of the buying organization.

Petersen, Frayer, & Scannell, (2000) - The ultimate purpose of supplier selection is to develop and maintain a competitive advantage in the marketplace. To successfully compete in the global marketplace of today, firms must meet or exceed the pace of rapidly changing technology while also lowering costs, increasing quality, and improving customer service at all stages of the value chain. For this purpose the technology involved in a firm can have a significant impact on competition

Zadeh (1965) initiated the fuzzy set theory and suggested the concept of fuzzy sets as a possible way of improving the modeling of vague parameters. Bellman and Zadeh (1970) suggested fuzzy programming model for decisions in fuzzy environment. Zimmermann (1976, 1978) presented a fuzzy optimization technique to linear programming (LP) problem with single and multiple objectives. Later, a number of researchers developed and applied fuzzy theories in the various areas of engineering application, such as, artificial intelligence, robotics, image processing, pattern recognition etc.

Dickson's work - a benchmark in the supplier's selection

1. Price/cost	13. Customer service
2. Delivery	14. Repair service
3. Warranties and claims	15. Training aids
4. Financial position	16. Geographical location
5. Operating controls	17. Performance history
6. Production facilities	18. Reputation and position
7. Technical capability	19. Amount of past business
8. Packaging ability	20. Labour relations record
9. Procedural compliance	21. Attitude
10. Management and organization	22. Impression
11. Communication system	23. Reciprocal arrangements
12. Desire for business	

Kumar, Vrat and Shankar (2004). In designing a supply chain, decision makers are attempting to involve strategic alliances with the potential vendors. Hence, vendor selection is a vital strategic issue for evolving an effective supply chain and the right vendors play a significant role in deciding the overall performance

Hannan, (1981) Yager, (1977) Narsimhan (1980) proposed a fuzzy goal programming(FGP) technique to specify imprecise aspiration levels of the fuzzy goals. **Yang, Ignizio, and Kim (1991)** formulated the FGP with non-linear membership functions

Objective

A lot of research work has been done in the field of selection, monitoring and maintaining appropriate vendors. But only little work has been done in allocating optimum quota to selected optimum number of vendors and that too in an uncertain environment with vague information. Now any vendor selection and quota allocation problem can be considered as a multi-objective problem with continuous variable in a fuzzy environment followed by its comparison with deterministic situation.

Working Steps of Calculations and Applications:

Input information collection and primitive comparative analysis

Table 4.1 : Vendor Source Data

Supplier No.	pi (Rs./ton)	qi (%)	li (%)	Ui (tons / year)	ri	fi	Bi (Million Rs / year.)
S1.	40,000	**0.02**	0.050	**6,250**	**0.97**	0.04	**2500**
S2.	33,000	0.08	**0.034**	3,000	0.90	0.03	1000
S3.	35,000	0.05	0.089	5,000	0.89	0.08	2000
S4.	**32,000**	0.10	0.045	2,000	0.79	**0.01**	600

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Formulation of multi-objective SQA Problem

MINIMISE 40000X1+ 33000X2 + 35000X3 + 32000X4	
MINIMISE 0.02X1 + 0.08X2 + 0.05X3 + 0.10X4	
MINIMISE 0.050X1 + 0.034X2 + 0.089X3 + 0.045X4	
SUBJECT TO :	
X1 + X2 + X3 + X4	= 12000
X1	<=6250
X2	<=3000
X3	<=5000
X4	<=2000
0.97X1+0.90X2+0.89X3+0.79X4	>=10920
0.04X1+0.03X2+0.08X3+0.01X4	<=600
40X1	<=2500000
33X2	<=1000000
35X3	<=2000000
32X4	<= 600000
X1	<=0
X2	<=0
X3	<=0
X4	<=0
END	

Table 4.2 : Primitive Comparative Analysis

Supplier No.	Comparative Comments from buyer's point of view
S1.	Best Quality but Highest (or worst) price. Also highest capacity to supply & highest budget allocation. The best vendor rating.
S2.	Best delivery performance, 2nd best price but quality is moderately poor. Second best vendor rating and flexibility.
S3.	Lowest or poor flexibility, 2nd best quality, 2nd highest budget to supply and 2nd highest vendor rating but 2nd highest (poor) price.
S4.	Lowest (the best) Price and the best flexibility but low capacity to supply and poor vendor rating.

FP (Fuzzy Programming)

In traditional multiple-objective optimization all the information known with certainty and preciseness. But in the real-life situation for a Supplier Selection Problem, many input information related to the various vendors are not known with certainty. Due to this, here a fuzzy model has been considered.

Modeling the VSP problem with fuzzy parameters

[LPP]	[fuzzy -LPP]
Minimize Z = Cx	
Subject to	~ Cx <~ Z0,
Ax = b,	~Ax <~ b,
x = 0.	x=0

The symbol '<~' in the constraint set denotes 'essentially smaller than or equal to' and allows one reach some aspiration level where, ~ C and ~ A represent fuzzy values.

Membership Function

In solving the [fuzzy-SQA] model, a linear membership function has been considered in this work for all fuzzy parameters. A linear membership function has a continuously increasing or decreasing value over the range of parameter. It is defined by the lower and upper values of the acceptability for that parameter.

A fuzzy objective ~ Z e X is a fuzzy subset of X characterized by its membership function

μZ (x): x . [0, 1]: The linear membership function for the fuzzy objectives is given as

$$\mu_{Zl}(x) = \begin{cases} 1 & \text{if } Zl(x) \leq Zl_{min}, \\ [Zl_{max} - Zl(x)] / [Zl_{max} - Zl_{min}] & \text{if } Zl_{min} \leq Zl(x) \leq Zl_{max}, \\ 0 & \text{if } Zl(x) \geq Zl_{max} \end{cases}$$

where l = 1, 2, . . ., L

Fuzzy Solution

In fuzzy programming modeling, using Bellman and Zadeh (1970) approach, a fuzzy solution is given by the intersection of the all the fuzzy sets representing either fuzzy objectives or fuzzy constraints. The membership function of the fuzzy solution is given by:

μS(x) = μZ(x) n μC(x) = min [μZ(x); μC(x)]

μC(x) and μS(x) represent the membership function of objectives, constraints and solutions, respectively.

The fuzzy solution of the [f-VSP] model for the L fuzzy multiple objectives and K fuzzy constraints may be given as

$$\mu S(x) = \left(\bigcap_{l=1}^{L} \mu Z(x) \right) \cap \left(\bigcap_{k=1}^{K} \mu C(x) \right) = \min \left[\min_{l=1;2;\ldots;L} \mu Zl(x) \; , \; \min_{k=1;2;\ldots;K} \mu Ck(x) \right]$$

An optimum solution of the [f-VSP] is one which has the highest degree of the membership value:

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Scientific Journal Impact Factor: 3.449
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Crisp formulation

Maximize γ
Subject to
$[\gamma\,(Zl_{max} - Zl_{min})] + [Zl(x)] \leq Zl_{max}$
$[\gamma\,(dxi) + gk(xi)] \leq [bk + dk]$
$Axi \leq b$
$xi \geq 0$
$0 \leq \gamma \leq 1$

for all l ; l = 1, 2, . . . , L ;
for all k; k = 1, 2, . . . , K;
All deterministic constant

The lower bound of the optimal values (Zlmin) is obtained by solving the VSP as a linear programming problem using each time only one of the objectives (ignoring all others):
Minimize Zl (xi) for all l ; l = 1, 2, . . . , L ;
Subject to gk(xi) = bk + d for all k ; k = 1, 2, . . . ,K ;
Axi = b for all deterministic constraints; xi =0
The upper bound of the optimal values (Zlmax) is obtained by solving a similar VSP as a linear programming problem:
Maximize Zl (xi) for all l; l = 1, 2. . . L;
Subject to gk(xi) = bk + dk for all k; k = 1, 2, . . . ,K
Axi = b for all deterministic constraints; xi =0

Application of [fuzzy -LPP] model to the SQA Model
The decision-maker's ambiguity about the fuzzy information related to the net cost, rejections, late deliveries and vendors' capacities is captured by applying the [fuzzy -LPP] model to the SQA model and fuzzified form [fuzzy -SQA] is expressed

[Fuzzy -SQA] Model

$$\sum_{i=1}^{n} pi(xi) \lesssim Z1,$$
$$\sum_{i=1}^{n} qi(xi) \lesssim Z2,$$
$$\sum_{i=1}^{n} li(xi) \lesssim Z3,$$
$$\sum_{i=1}^{n} xi = D,$$
$$xi \lesssim Ui,$$
$$\sum_{i=1}^{n} ri(xi) \geq P,$$
$$\sum_{i=1}^{n} f\,i(xi) \leq F,$$
$$pi(xi) \lesssim Bi,$$
$$xi \geq 0$$

Application of Fuzzy Programming (FP) to case study SQA problem with fuzziness captured in capacities and allocated budgets of suppliers:
According to the theory, the linear membership function is used for fuzzifying the right-hand side of the constraints in the basic SQA model. The values of the level of uncertainties for all the fuzzy parameters are taken in incremental steps of 5% of the deterministic model. In order to find the values of cost, rejections, late deliveries at the lowest and highest level of membership function. The model is solved using LINDO – with first minimization and then maximization of only one objective at a time, ignoring all the other objectives. The data set for the values at the lowest and highest aspiration levels of the membership functions with 5% uncertainty solution is given by the intersection of the all the

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Scientific Journal Impact Factor: 3.449
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Formulation of SQA Problem with 5% increase in vendor's capacities and budget.

```
MINIMISE or MAXIMISE   40000X1+ 33000X2 + 35000X3 + 32000X4
MINIMISE or MAXIMISE   0.02X1 + 0.08X2 + 0.05X3 + 0.10X4
MINIMISE or MAXIMISE   0.050X1 + 0.034X2 + 0.089X3 + 0.045X4
SUBJECT TO :
X1 + X2 + X3 + X4                 = 12000
X1                                <= 6562.5
X2                                <= 3150
X3                                <= 5250
X4                                <= 2100
0.97X1+0.90X2+0.89X3+0.79X4       >= 10920
0.04X1+0.03X2+0.08X3+0.01X4       <= 600
40000X1                           <= 2625000000
33000X2                           <= 1050000000
35000X3                           <= 2100000000
32000X4                           <= 630000000
X1, X2, X3, X4                    <= 0
END
```

Limiting values in linear membership functions of objectives with 5% uncertainty in supplier's capacities and budget.

Objectives	Minimum Limit below which γ=1	Maximum Limit above which γ=0	Difference
Net cost objective	429,392,200	448,912,500	19,520,300
Rejection objective	501.563	596.063	94.500
Late deliveries objective	552.188	725.438	173.250
Capacity Constraints			
Supplier S1	6250	6562.5	312.5
Supplier S2	3000	3150	150
Supplier S3	5000	5250	250
Supplier S4	2000	2100	100
Budget Constraints			
Supplier S1	250,000,000	262500000	12,500,000
Supplier S2	100,000,000	105000000	5,000,000
Supplier S3	200,000,000	210000000	10,000,000
Supplier S4	60,000,000	63000000	3,000,000

Applying the results and the concepts of **[crisp -LPP] Model** to the adopted SQA problem at 5% degree of uncertainty, a crisp fuzzy linear programming is formulated. In this model, the aim is to find the optimal quota allocation to the suppliers while maximizing. (the degree of overall satisfaction).

A crisp-LPP formulation for case study SQA problem with 5% fuzziness about Vendor's capacities and budget.

Model 4

```
MAXIMISE  γ

Subject To :

19520300γ +40000x1+33000x2+35000x3+32000x4 <= 448912000
94.5γ +0.02X1 + 0.08X2 + 0.05X3 + 0.10X4    <= 596.063
173.25γ +0.050X1 + 0.034X2 + 0.089X3 + 0.045X4 <= 725.438
X1 + X2 + X3 + X4                            = 12000
312.5γ +X1                                   <= 6562.5
150γ +X2                                     <= 3150
250γ +X3                                     <= 5250
100γ +X4                                     <= 2100
0.97X1+0.90X2+0.89X3+0.79X4                  >= 10920
0.04X1+0.03X2+0.08X3+0.01X4                  <= 600
12500000γ +40000X1                           <= 262500000
5000000γ +33000X2                            <= 105000000
10000000γ +35000X3                           <= 210000000
3000000γ +32000X4                            <= 63000000
END
```

Result analysis

Here the maximum degree of overall satisfaction (.max) of 0.15 only is achieved for the solution S1 = 5268; S2 = 1404 ; S3 = 3374 and S4 = 1955. This provides the best solution at 5% uncertainty for an aggregate demand of 12000 tons, yielding Rs 437.7 millions as the net cost, 582 tons as total number of rejections and 699 tons as total number of late delivered items. In this fuzzy formulation, the quota allocations of suppliers 1 and 4 have increased by 41.6% and 123.4% as compared to the deterministic model, whereas the quota of vendor 2 and 3 has decreased by 53.2% and 23.4% as compared to the deterministic model. This is due to 5% vagueness that has been captured in the estimate of vendors' capacities and budgets.

Results of application of models 3 & 4 to SQA problem with 5% to 50% vagueness created in vendor's capacities and budgets

% of Uncertainty	Degree of Satisfaction	S1	S2	S3	S4	Total Cost (Rs.)	Rejection	Late deliveries
5%	0.151	5268	1404	3374	1955	437,666,702	581.8	699.3
10%	0.163	6773	3251	1976	0	447,363,480	494.3	625.0
15%	0.190	70[illegible]	3364	1627	0	448,316,440	490.7	609.6
20%	0.214	7232	3471	1297	0	449,217,300	487.2	595.0
25%	0.236	7444	3573	982	0	450,074,980	483.9	581.1
30%	0.190	8108	3892	0	0	452,756,770	473.5	537.7
35%	0.181	8108	3892	0	0	452,756,770	473.5	537.7
40%	0.179	8304	3696	0	0	454,124,964	461.8	540.9
45%	0.175	8571	3429	0	0	455,998,120	445.7	545.1
50%	0.165	8860	3140	0	0	458,022,100	428.4	549.8

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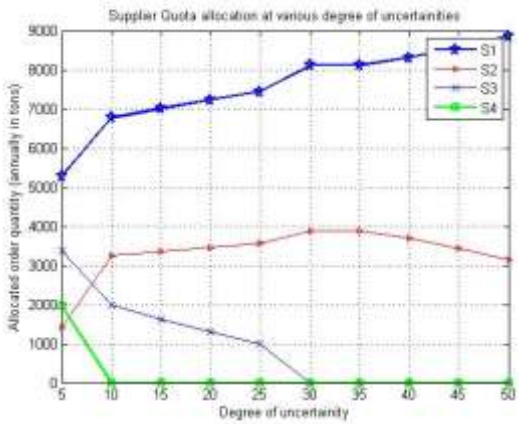
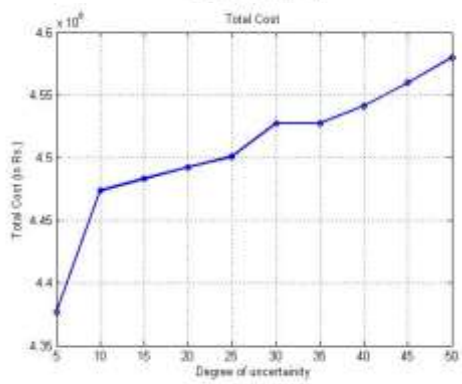


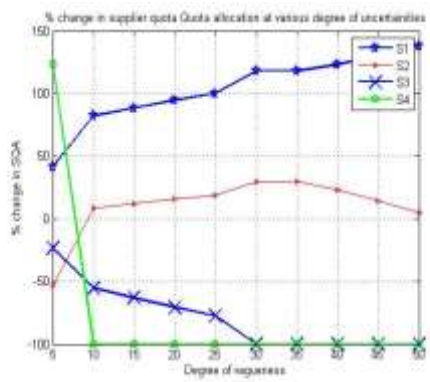


The above graphs clearly reveal that with the increase in degree of uncertainty / fuzziness about vendor's deterministic parameters from 5% to 50%, the net cost of allocating 12000 tons to 4 vendors increases continuously while the rejections and late delivered items decrease with overall degree of satisfaction varying in a range of 0.15 to 0.25.

% change in quota allocations and objectives with 5% to 50% change in degree of uncertainty by FP.

	Base	5%	10%	15%	20%
S1	3719	41.6%	82.1%	88.5%	94.5%
S2	3000	-53.2%	8.4%	12.1%	15.7%
S3	4406	-23.4%	-55.2%	-63.1%	-70.6%
S4	875	123.4%	-100.0%	-100.0%	-100.0%
Cost	430000000	1.8%	4.0%	4.3%	4.5%
Rejections	478.5	21.6%	3.3%	2.5%	1.8%
Late Deliveries	576.75	21.3%	8.4%	5.7%	3.2%

	25%	30%	35%	40%	45%	50%
S1	100.2%	118.0%	118.0%	123.3%	130.5%	138.2%
S2	19.1%	29.7%	29.7%	23.2%	14.3%	4.7%
S3	-77.7%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%
S4	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%
Cost	4.7%	5.3%	5.3%	5.6%	6.0%	6.5%
Rejections	1.1%	-1.0%	-1.0%	-3.5%	-6.8%	-10.5%
Late Deliveries	0.8%	-6.8%	-6.8%	-6.2%	-5.5%	-4.7%



Results & conclusions

FP (Fuzzy Programming)

% of uncertainty	S1	S2	S3	S4
5%	5268	1404	3374	1955
10%	6773	3251	1976	0
15%	7009	3364	1627	0
20%	7232	3471	1297	0
25%	7444	3573	982	0
30%	8108	3892	0	0
35%	8108	3892	0	0
40%	8304	3696	0	0
45%	8571	3429	0	0
50%	8860	3140	0	0

.FP (Fuzzy Programming)

% of uncertainty	Cost	Rejection	Late Del.	Degree of Satisf.
5%	437,666,702	581.8	699.3	0.151
10%	447,363,480	494.3	625.0	0.163
15%	448,316,440	490.7	609.6	0.190
20%	449,217,300	487.2	595.0	0.214
25%	450,074,980	483.9	581.1	0.236
30%	452,756,770	473.5	537.7	0.190
35%	452,756,770	473.5	537.7	0.181
40%	454,124,964	461.8	540.9	0.179
45%	455,998,120	445.7	545.1	0.175
50%	458,022,100	428.4	549.8	0.165

In fuzzy programming model, an attempt is made to determine the vendor's quota in a supply chain when various parameters of vendors are not

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

known with certainty. The advantages of the fuzzy modeling is that the complexity of the vendor quota allocation problem may be handled even if the capacity of each vendor is vague, which may be due to limited sharing of internal data between buyer and supplier.

Suggestions / recommendation

1. **S1** is the most preferred supplier in the approved vendor's list of the firm because as compared to other three vendors his:
   - ✓ Vendor rating of 97% is the highest.
   - ✓ Quality is the best (as % rejection is only 0.02%).
   - ✓ Capacity to supply is maximum.
   - ✓ Budget to supply is enormous.
2. **S4** has quoted the lowest price of Rs 32000 per ton but his vendor rating is very poor and he has low capacity to supply. The most negative aspect of S4 is the high rejection rate of 0.10%, which may perhaps be the reason for his low pricing.
3. **S2** in his favour has best performance in on-time delivery of items to the company, which is commendable. Most of his other performance criteria are always of second degree in comparison to other vendors.
   - ✓ His quoted price of Rs. 33000 per ton is only one thousand rupees more than S4.
   - ✓ His flexibility in supply of items is also second best after S4.
   - ✓ The vendor rating of 0.90% of S2 is second best only after S1.
   - ✓ But his poor quality of product (with rejection level of 0.08%) is only better than S4– again second from bottom. Also this poor quality is a major contributor in lowering his vendor rating.
4. **S3** has none of the performance criteria in his favour. He is either moderately good or moderately poor.
   - ✓ In slightly favourable position S3 has rejection level of 0.05% which makes him better placed after S1in supplying good quality material. Also S3 has second highest capacity to supply and budget allocation after S1.
   - ✓ On the other hand in slightly unfavourable position S3 has 0.089% record of delivering late items. He is the mainly responsible for non-achievement of on-time delivery objective of the organisation. Also S3 has poor flexibility along with second highest price of RS 35000 per ton. after S1.

Finally based on the study, the preference order of selection of vendors that could be considered for optimal quota allocation is:
S1 > S2 > S3 > S4.

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ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

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